INVESTMENT IMPLICATIONS OF THE ACTUARIAL DESIGN OF LIFE INSURANCE PRODUCTS

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ABSTRACT

The traditional design of life insurance products marketed in the United States has included level premiums graded by initial age, fixed dollar benefits, and relatively high withdrawal benefits. This design has certain implications on life insurance investments. Because of the long term and fixed dollar nature of life insurance liabilities, investments with similar characteristics are indicated. In recent years, marked with both price inflation and real economic growth, the purchasers of the products and services of life insurance companies have seemed to place less value on investment guarantees. A new family of products and services is being developed that will cause a shift in the investment objectives of life insurance companies.

Augustus de Morgan (1808-1871), British mathematician, actuary, logician, and precursor of the modern subjective school of probability, once remarked, "It may safely be said there is nothing in the commercial world which approaches, even remotely, the security of a well established and prudently managed insurance office."

De Morgan made this remark during the middle of the Victorian age, an epoch blessed to a singular degree with stable prices and interest rates [24, Chart 1]. Not only were economic conditions relatively constant, but life insurance offices found mortality trends to be running in their favor. Actuaries had already constructed the fundamental plan for individual life insurance which provides for level premiums graded by age, and the accumulation and investment of the funds, built up as a consequence of the fact that for a block of similar life insurance policies the bulk of the distribution of the time of receipt of the premium income is to the left of the distribution of the time of disbursement of benefit payments. This elegant plan for the orderly accumulation, investment, and ultimate disbursement of funds undoubtedly deserved de Morgan's compliment. However, the stability that de Morgan praised may be destroyed by the failure of the invested funds to earn interest income at a rate anticipated in the premium structure, by capital losses, and by extraordinary benefit payments.

The successful implementation of the actuarial master plan for individual life insurance and for both group and individual annuity systems requires that the investment cash flows be predictable and that investments be managed so that these predictions are at least approximately realized. Therefore, because the management of the investments of a life insurance system must be directed so that they at least achieve the results required by the price-benefit structure developed by an application of actuarial theory, it is natu-

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ral that the theory of insurance investment management and actuarial science should intersect.

During the nineteenth century the main ideas of life insurance actuarial science were being developed and codified. The principal project in this endeavor was in blending ideas from probability and from the mathematics of compound interest to form the mathematics of life contingencies. An ancillary part of the work was to systematize the mathematics of compound interest and to relate these ideas to the practical investment problems faced by life office managers. The book by Ralph Todhunter [31] will give present day students an idea of the intersection between actuarial science and the mathematical aspects of the theory of finance as defined at the end of the nineteenth century.

The two central problems in the theory of compound interest are (1) to value a given stream of payments at a fixed interest rate, and (2) to determine the internal rate(s) of return defined by a known stream of payments. To illustrate the solutions to both types of problems, Todhunter often used examples provided by securities available to British life offices at the time he wrote.

Todhunter's text may seem to the present day students to consist of a collection of tricks for solving perplexing but fundamentally simple mathematics problems. However, Todhunter was not unaware that these tricks were relevant to real investment management problems. For example, in recent years the subject of the relationship between the internal rate of return in common stock investments and the dividend growth rate has attracted some attention in finance. Soldofsky [29] has pointed out that not only had Todhunter solved the simple problem of summing the infinite geometric progression that results when converting a stream of future geometrically increasing dividends to a present value, but he had provided an interpretation of his answer to this problem.

Yet it would be fair to judge that, except for the theory of immunization, which will be discussed later in this essay, the actuarial literature for the first six decades of this century does not contain extensive discussions of new ideas on the management of investments. Perhaps this is due to the division of labor, especially in the United States, between actuarial departments and investment departments within life insurance offices and between the consulting actuary and the pension fund trustee in pension systems. Another possible explanation is that perhaps the actuarial and financial theory needed to successfully manage a life insurance company and a pension fund, at least during periods with stable prices and interest rates, is essentially complete and no additional theoretical construction is required.

This rather arbitrary and negative judgment about the amount of activity within the intersection of actuarial science and finance during the first part of this century was designed to exclude the recent work on equity linked life insurance and annuity contracts [6, 12, 13]. The design of this type of policy requires the solution of extremely interesting actuarial problems. When minimum benefit guarantees are also introduced, a unique type of risk results which has required the creation of a new methodology for measuring and managing the risk [9, 17, 32]. In essence, a new master plan for individual life insurance, which incorporates many ideas from the theory of finance, is being developed.

In North America actuarial science has intersected with finance on three topics: (1) the estimation of the interest rate(s) that may safely be used in determining pension and life insurance price-benefit structures, [10], (2) the determination of the mix of investment policy, level of cash value guarantees, and delay provisions in life insurance contracts which will minimize the threat to solvency posed by the existence of guaranteed cash and loan values [21, 28], and (3) the problem of assigning values to assets for accounting statement purposes [25]. At first glance the third topic does not seem to deserve the attention that it has received in North American insurance and actuarial journals.

The prominence of the third topic may be attributed to the rigidity imposed by a political decision made in the United States to require guaranteed cash values and to compel net premium reserves coupled with legally constrained valuation assumptions rather than flexible cash values, gross premium reserves, and currently realistic actuarial assumptions. This decision has imposed a degree of woodenness in the valuation of liabilities of United States life insurance companies.

To avoid technical insolvency, which might result from local aberrations in the securities markets, if asset values were linked to market values and liabilities were fixed, a corresponding stability is required in valuing the assets of life insurance companies. This requirement explains the large amount of time spent on the question of assigning accounting values to assets and to the associated topic, the Mandatory Security Valuation Reserve.

The economic conditions that have prevailed in the last few years of the decade of the sixties illustrate the practical necessity to shelter insurance asset accounting values from current market realities, given our fixed liability valuation system. In the past few years life insurance companies have valued most of their statutory liabilities, and computed the associated guaranteed cash values, at interest assumptions between 2½ and 3½ percent. At the same time, bonds have been selling at prices to yield at rates two and three times these valuation rates. The result has been depressed bond prices, depressed to a state such that if bond market values were used in insurance accounting, many insurance companies would find their surplus at apparently dangerously low levels.

Given the political realities under which insurance regulation developed in the United States, this family of regulatory decisions about life insurance asset and liability values may have been the only member of the set of feasible families of decisions. At the time that these decisions were effectively made, insurance regulation was completely in the hands of state agencies that individually had difficulty in commanding the resources to employ the technical experts necessary to intelligently administer a flexible regulation program.

Perhaps all that can be said now is that the regulation system that evolved in the United States includes objective, but somewhat artificial standards, for valuing life insurance assets and liabilities. These standards have perplexed those who encounter insurance accounting statements for the first time and they probably have impeded the response of United States life insurance companies to changing social and economic conditions. When compared to the more flexible and informal system of regulation in Great Britain, it is hard to build a case for the superior performance of the United States type of regulation in protecting the public interest. However, to speculate on whether a more flexible type of regulation would have in fact promoted the general welfare, given the state of business ethics, the public philosophy, and the number and distribution of actuaries in the United States, is a fascinating exercise but one which can never lead to a definite conclusion.

It is not the purpose of this essay to explore the regulation of life insurance asset and liability values. However, it is difficult to leave this topic without pointing out the remarkably different systems that have developed in the United States for regulating life insurance companies and noninsured pension systems. It is acknowledged that the contracts that formalize pension and insurance systems are fundamentally different. Nevertheless, it would appear that the fulfillment of the reasonable expectations of participants in both insurance and pension systems would be in the public interest.

Yet the principal agency regulating noninsured pension systems is a federal agency, the Internal Revenue Service (I.R.S.), while insurance is regulated by fifty-one different state agencies. The primary objective of the I.R.S. is to limit business expense deductions for pension purposes. Consistent with this objective the I.R.S. permits a variety of funding methods, pushes for valuation assumptions that relate to current experience, and encourages valuation methods that anticipate gains and losses due to withdrawals.

Insurance regulation, directed to providing simple, objective tests of solvency, has prescribed conservative valuation assumptions that are invariant over time, and net premium valuation methods, with the resulting implicit assumption that there will be no future gains or losses from withdrawals or from differences between premium loadings and expenses. It would be an interesting project in the history of business regulation to trace the twin chains of events that have produced such different regulatory systems for related private enterprises.

Investment Objectives

Some insight into the relationship between the actuarial model for individual life insurance and the associated investment objectives of a life insurance office may be gained by referring to two papers in the actuarial literature which were written over a century apart but which develop strikingly similar ideas. Writing in 1862 A. H. Bailey [4] stated that the investment objectives of a life insurance office should be:

1. Security of capital.
2. Highest practicable rate of interest, subordinate to security.
3. Maintain a small proportion of funds in convertible securities to meet claims and loan demands.
4. Maintain the largest proportion of funds in securities which are not readily convertible. These should command a higher rate of return because they are unsuited for private individuals.
5. Use investments to promote the insurance industry.

In 1967 Houghton and Farmer [18] took their turn at listing the objectives of life insurance investment that they viewed as being consistent with the actuarial master plan. Houghton and Farmer state that investment policy should seek the following goals:

1. high yield
2. capital gains
3. useful byproducts.

The attainment of these goals is to be subject to the following constraints:

1. safety of principal
2. liquidity as required to meet unexpected demands
3. stability of asset values
4. ease of ascertaining asset values
5. economy of obtaining, maintaining, and disposing of the investment
6. legality
7. tax and Mandatory Security Valuation Reserve considerations.

Houghton and Farmer were offering advice to the management of new life insurance companies doing business in the United States. This accounts for the prominence given to points 3, 4, 6 and 7 in the constraining conditions. After making allowance for this fact, the similarity between these two lists of investment objectives is remarkable.

It would be a mistake to assume that because of the essential agreement between these two lists of investment objectives, there is an enduring concensus on this subject. In particular, the ranking that these lists attach to liquidity has been questioned. Even in the depths of the Great Depression the annual cash flows through the 20 largest United States life insurance companies were positive [22]. Therefore, many have questioned whether any weight should be given to liquidity as an investment objective.

Matching

The primary evidence that lack of liquidity, and the related absence of the security of capital, are not overriding threats to well managed life insurance companies, is the experience of the Great Depression. However, the experience of both British and North American companies in the immediate post World War II years indicate that the threat of inadequate investment income can be very serious, [7, Chap. 4]. In both Great Britain and the United States, it was government policy to hold interest rates artifically low during that period. The impact of this policy on life insurance companies is easy to observe. A glance down the column of industry investment yield rates during the late 1940's will indicate that these rates were below the 3 and 3½ percent interest rates that were used to calculate most reserves and, worse yet, to determine most cash values (2.88 percent in 1947, [20, p. 60]).

The late W. M. Anderson, Chairman of North America Life Assurance Company [2] put the case very forcefully when he stated, "The major risk facing a life insurance company is a change in the riskless interest rate." Life insurance companies are particularly exposed to this risk because the funds necessary to fulfill existing contracts will become available for investment or reinvestment in the future at presently unknown rates. The dominance of this risk in the eyes of management cannot help but discourage actuaries who have developed an elaborate theory to measure and manage risk created by the random nature of benefit payments. Yet, according to Anderson, and his view is shared by many insurance managers, random variation in claims payments is not the primary problem.

In 1952 and 1953 three remarkable papers [3, 16, 27] were published in British actuarial journals concerning investment management plans for reducing the adverse effect on the balance between the assets and liabilities of a life insurance office caused by interest rate changes. It is ironic that these ideas were being developed concurrently with the turn around in interest rates which had the impact of reducing the urgency to implement these ideas.

The key idea developed in these papers is that the investment of life insurance funds can be arranged with respect to the dates of receipt of both capital repayments and interest income so that the balance between the expected income and the expected outgo for existing policies will not be destroyed by a change in the interest rate. The idea of matching the liabilities due in one currency with assets valued in terms of the same currency is an old one. What was being called for at that time was a matching of the time incidence of insurance and investment cash flows. To make a distinction between the new process, which was designed to protect against interest rate changes, and the older matching process which relates to currency matching, Redington [27] suggested the word "immunization" to describe the time matching process.

Although the development has appeared in many other places, for completeness it seems appropriate to show the two simple immunization rules of Redington. We let $i(t)$ denote the expected net insurance cash flow in year t in the future which for convenience will be assumed massed at the end of the year. This flow is made up of benefit payments and expenses less premium income for the block of policies being immunized. We let $a(t)$ denote the expected investment cash flow, interest and capital repayments, occurring in year t, which for convenience will be assumed to be massed at the end of the year. These asset payments are those from the assets assigned to the block of insurance policies that generate the sequence $i(t)$, $t = 1, 2, \ldots,$ of insurance payments.

Then the present expected value of the insurance stream of payments is given by $I(r) = \sum_{t=1}^{\infty} v^t i(t)$, and the present expected value of the asset stream of payments is given by $A(r) = \sum_{t=1}^{\infty} v^t a(t)$, where $v = (1+r)^{-1}$, and r is the annual effective interest rate considered appropriate as a realistic valuation assumption at the time that the valuation is performed. We shall assume that at this initial time $I(r) = A(r)$. If $A(r) > I(r)$, then the excess assets may be invested free of the immunization rules. If $A(r) < I(r)$, then additional assets will have to be allocated from surplus to match the existing insurance liabilities.

The question in which we are interested is the impact of an immediate change in the interest rate from r to $r+\Delta$, where Δ may be negative, on the balance between $I(r+\Delta)$ and $A(r+\Delta)$. To study this question we display the Taylor series expansion of $A(r+\Delta) - I(r+\Delta)$ about the point r.

$$A(r+\Delta) - I(r+\Delta) = [A(r) - I(r)] + \Delta[A'(r) - I'(r)] + \Delta^2[A''(\theta) - I''(\theta)]/2,$$

where θ is between r and $r+\Delta$. We seek to insure that $A(r+\Delta) - I(r+\Delta) \geqq 0$. Since $A(r) - I(r) = 0$, we see that our objective may be achieved if (1) $A'(r) - I'(r) = 0$ and (2) $A''(\theta) - I''(\theta) \geqq 0$. These conditions may be written, after a few manipulations, as follows:

$$(1) \quad \sum_{t=1}^{\infty} tv^t a(t) = \sum_{t=1}^{\infty} tv^t i(t), \quad \text{where } v = (1+r)^{-1},$$

and

$$(2) \quad \sum_{t=1}^{\infty} t^2 v^t a(t) \geq \sum_{t=1}^{\infty} t^2 v^t i(t), \quad \text{where } v = (1+\theta)^{-1}.$$

Because θ is unknown, we shall substitute (2)′ as a practical operating approximation for (2).

$$(2)' \quad \sum_{t=1}^{\infty} t^2 v^t a(t) \geq \sum t^2 v^t i(t), \quad \text{where } v = (1+r)^{-1}.$$

The operating immunization rules (1) and (2)′ may be stated in statistical terms. We define two discrete "mass" functions $f(t) = v^t a(t)/A(r)$ and $g(t) = v^t i(t)/I(r)$. Then (1) and (2)′ combined simply require that the mean of the two "distributions" be identical, but that the variance of the "distribution" with mass function $f(t)$ be greater than the variance of the "distribution" with mass function $g(t)$. Crudely, the rules simply state that to reduce the impact on the balance between insurance and asset cash flows, if the interest rate changes, invest long. The "distribution" of the asset payments is to have the same mean time of receipt but the payments should be more dispersed than the insurance payments. In this paragraph the word "distribution" has been placed in sanitary quotation marks because $g(t)$ may not be a mass function since some values of $i(t)$ may be negative. In addition, since the sequences $[a(t)]$ and $[i(t)]$, are already composed of expected values, there is no ready probability interpretation of these two "distributions."

For the first half of the decade of the fifties, within British actuarial circles a great deal of effort was directed to refining and specializing the immunization idea and to understanding the practical problems in implementing the rules [11, 30]. Despite this work, it would be a mistake to assume that the immunization rules ever became a powerful consideration in life insurance investment in Great Britain and they had even less impact in the United States.

The most forceful reason preventing life insurance companies from embarking on an immunization program was the fact that immunization is (borrowing Redington's colorful phrase) "a two edged sword"; it immunizes against gains as well as against losses due to interest rate changes. At about the time that the immunization theory was completed, interest rates started a climb that continued for almost 20 years and no one wants to be protected against gains.

Anderson [2] believed that North American insurance investment officers could afford to ignore the immunization rules because their traditional reaction to interest rate changes tends to beat the performance of immunization. That is, in times of low interest rates, the life insurance companies have shifted from long term bonds to mortgages with their shorter average term. In periods of high interest rates, the life companies have tended to shift from mortgages to long term bonds. Brimmer [5, p. 103] also commented on this policy. The objective of this policy is to dull the edge of the immunization sword which reduces gains when interest rates turn up.

Besides these broad business considerations that have reduced the practical impact of the immunization idea, there are some technical problems that hamper its adoption. The first, and perhaps the most obvious, is the uncertainty surrounding the computation. The sequence of insurance payments [i(t)] was defined to consist of expected values. Despite the relative stability of claims payments and premium income for large blocks of policies, withdrawal benefits may depend on unpredictable economic conditions and therefore, behave in a capricious fashion and invalidate the distribution assumptions made in computing the sequence [i(t)]. A change in the rate of election of income settlement options could also profoundly effect the distribution of insurance payments.

Uncertainty in investment payments caused by bond call provisions and other optional features, which are increasingly common in securities, may be quantified in the form of a distribution of subjective probability for the purpose of estimating the sequence of expected investment income payments. However, because of the possible irrelevance of past experience in estimating these distributions, they might be rather diffuse and at best could represent only the business judgment of the investment officers. In summary, immunization can be achieved by using expected cash flows but the probability that these flows will be achieved is low.

Another rather technical consideration, but one which has very practical implications that restrict the implementation of the immunization idea, is that the weighted average time of the insurance payments is very long for life insurance. So long in fact, that it is difficult in most practical situations to reach an immunized position. Table 1 is directly related to a similar table in a paper by Wallas [34]. The table gives formulas for computing the mean time for the insurance payments and a numerical example. The example is concerned with the immunization of a block of whole life policies of amount one, issued at age 40, assuming, rather arbitrarily that the gross premium less ex-

TABLE 1

IMMUNIZATION TABLE, 1958 CSO, 3% INTEREST

Annual Rate of Increase in Entrants	*s = 0*	*s = .05*	*s = .10*
	Sums Assured		
Amount			
$\int_0^\infty {}_tP_{40}(1+s)^{-t}dt$	32.179	15.379	9.505
Present Value			
$\int_0^\infty {}_tP_{40}(1+s)^{-t}\bar{A}_{40+t}dt$	19.869	8.463	4.836
Mean Term			
$\dfrac{\int_0^\infty {}_tP_{40}(1+s)^{-t}(\bar{I}\bar{A})_{40+t}dt}{\text{(Present Value)}}$	13.918	17.143	19.210
	Premiums		
Amount			
$.02\int_0^\infty {}_tP_{40}(1+s)^{-t}dt$	0.642	0.307	0.190
Present Value			
$.02\int_0^\infty {}_tP_{40}(1+s)^{-t}\bar{a}_{40+t}dt$	8.956	4.977	3.343
Mean Term			
$\dfrac{.02\int_0^\infty {}_tP_{40}(1+s)^{-t}\bar{I}\bar{a}_{40+t}dt}{\text{(Present Value)}}$	10.420	11.334	11.759
	Net Insurance Liabilities		
Present Value (Sum assured-Premiums)	10.914	3.487	1.494
Mean Term #	16.788	25.435	35.888

Mr. Richard Maurer prepared Table 1 using the computing facilities of the University of Iowa. The trapezoidal rule was used to evaluate all integrals. The table originally appeared in a discussion of "A Logical Approach to Population Problems", by Robert W. Batten, *Transactions, Society of Actuaries*, Vol. 21 (1969) and appears with the permission of the Editor.

Mean Term Net Liabilities = [(Present Value Sum Assured) (Mean Term Sum Assured) − (Present Value Premiums) (Mean Term Premiums)]/[Present Value Sum Assured) − (Present Value Premiums)].

penses is the net premium. The computations were made assuming a continuous model. At first a stationary population was assumed and then populations of insured lives that have grown at annual rates of 5 and 10 percent were assumed. To simplify comparisons, the amount and present value results have been stated per life at the current rate of entry, that is expected payments have been computed using $l_{40+t}/l_{40} = {}_tp_{40}$.

Of course, as is usual in the application of stationary and dynamic population theory to actuarial problems, these results may be viewed only as illustrations. The practical problems in implementing a program of immunization and the cost of foregoing possible gains as a result of interest rate changes are ignored in the example. Nevertheless certain serious practical problems are evident even in this simple example. The mean time of the liability payments has been overstated by ignoring withdrawal payments and by an unrealistic expense assumption. However, it appears that, even after making mental adjustments for these biases, it would be difficult to plan a schedule of asset maturities which would produce a mean time of asset payments which would equal the mean time of liability payments.

The study of the mean time of receipt of the asset income stream had already been started, many years before the development of the immunization idea, by Macaulay [23]. Macaulay was studying the behavior of interest rates and bond yields and in the course of this study, he defined the concept of "duration" which is identical to the mean time of receipt idea expressed by the left hand member of equation (1). We will follow Macaulay and study a debt based security with present value denoted by $P(n,c,I,r) = Cv^n + Ia_{\overline{n}|}$, where C is the final payment at time n, I is the constant periodic payment paid with the same frequency as the interest rate r used in valuing $v = (1+r)^{-1}$ is converted, and $a_{\overline{n}|} = \sum_{1}^{n} v^t$. We let

$$D(n,c,I,r) = (nCv^n + I\sum_{1}^{n} tv^t)/P(n,c,I,r)$$

and call $D(n,c,I,r)$ the mean time or duration associated with the investment. This expression as it stands represents the mean time for a straight team bond where C is the redemption value and I the periodic coupon payment. Table 2 summarizes some properties of $D(n,c,I,r)$ for some common types of debt based securities.

TABLE 2

MEAN TIME CHARACTERISTICS OF COMMON DEBT SECURITIES

Specification	Name	D(n,c,I,r)
$c=0$	Amortizing Mortgage	$(1+r)/r-nv^n/(1-v^n)$
$n=\infty,\ c=0$	Consul	$(1+r)/r$
$I=0$	Savings bond	n

A glance at the mean term entries, especially the insurance mean times for growing companies, and a few trial values in the $D(n,C,I,r)$ column of Table 2 will indicate the problem of attaining an immunized position.

In the discussions of immunization in both Great Britain and the United States [33], it has been suggested that equity investments might provide a means for securing very large mean time of asset receipts to match the mean time for growing insurance liabilities. If it is assumed that the price for a equity investment is $P(D,j,r) = \sum_{t=1}^{\infty} D(1+j)^t\ v^t = (1+j)/(r-j)$, where $D(1+j)^t$ is the expected dividend at time t, it is easy to show that the mean time associated with this expected stream of investment payments is $(1+r)/(r-j)$, assuming $r > j$. This can, quite obviously, be very large in the case that j and r are close together. Of course the assumption that future dividends may be estimated by the expression

$D(1+j)^t$, $t=1, 2, \ldots$, sweeps away, with a simplistic assumption a very perplexing problem. This illustration stresses that in the real world immunization is achieved only in the expected value sense, not as a certainty.

This example would be more satisfactory to modern day students of finance if the expected return at time t, $D(1+j)^t$, was interpreted as the total expected income, expected dividend plus expected capital value change, associated with the ownership of this equity security. The realization of the expected cash flow of investment income might require a continuing program of stock sales.

Suppose that the dividend policy of the issuing corporation provides for retaining and reinvesting in the enterprise all earnings. Then, if the insurance company holds this stock for n years and sells, it, the expected cash flow will be $D(1+j)^n$ and the mean time will be $n[(1+j)/(1+r)]^n / [(1+j)/(r-j)]$. The realization of these results would depend, of course, on the success of the issuing corporation in employing retained earnings at the growth rate j within the corporation, and the recognition of this increase in value in the stock market.

In this section emphasis has been placed upon the actuarial contributions to the idea of time matching. However, one should be aware of the fact that the same idea has been discussed in the literature of business and economics [19, 35].

Decline of Guarantees

The theory of immunization was constructed as a response to the crunch put on life insurance managers caught between fixed unit liabilities, with built in interest rate guarantees, and interest rates on bonds held low by collusion between fiscal and monetary authorities. In part because of the technical and economic problems in implementing an immunization program, the idea never had great influence in North America. However, the principal reason for the failure of immunization to take hold, has been the economic conditions that have prevailed since the early 1950's.

Interest rates have increased during most of this period, prices edged upward, then leaped upward; but during most of the period personal income has kept ahead of price increases to propel living standards to new highs. In such a period, long term fixed dollar guarantees commenced to lose their appeal. Individual and corporate savers, as well as purchasers of death benefits and retirement income, became concerned with achieving goals not stated in terms of fixed dollars but in terms of living standards and the preservation of command over physical assets.

The products marketed in the past by United States life insurance companies, both to individual purchasers of life insurance and corporate purchasers of deferred group annuities, have possessed guaranteed benefit amounts. To match these guarantees, built into their liabilities, it was prudent for life insurance managers to invest in securities with the same attributes; that is with fixed interest and capital repayment schedules and long terms to maturity. The critics of life insurance investment who have claimed to have found a serious lack of audacity, may not have had a deep understanding of the nature of the liabilities that these investments were to match. Because of the relative high cash values granted in North American life insurance, when compared to those in Great Britain, investment offices on this side of the Atlantic have felt that they held less freedom of action than their British counterparts [26].

Many of the insurance product and service developments of the past decade have been a reflection of the reduced value attached to benefit guarantees. There is an apparent willingness on the part of purchasers of the services and products of life insurance companies to trade guarantees for an increased expectation that they will participate in growth of the economy. Examples of the impact of this willingness on insurance products and services are easy to find.

1. Variable, equity linked, life annuities. Historically, this product was the first of the new line. The plan for the variable annuity usually has the insurance company absorb the mortality and expense risk while investment performance is reflected directly in the benefit amounts [6, 12].

2. Variable, equity linked, life insurance. These products are still under development but, like the variable annuity, the plan calls for the insurance company to absorb the mortality and expense risk and to reflect more or less directly investment performance in benefit amounts paid [13].

3. Generation allocation of investment income [14]. This idea requires that each year the new money entering a life insurance company be identified as to the line of business that generated the positive cash flow. Investment income generated by the investments associated with this new money will, in subsequent years, be allocated by line of business, and by individual contract holders in group annuities, according to their proportionate share of the new money.

This idea was motivated by a desire to recapture pension assets that were moving to trustees who were offering more immediate participation in the increasing interest earnings. Considerations of equity, as well as the practical business requirement to discourage the exit of pension assets, may have dictated this change in the method of allocating investment income. Yet this approach does reduce the effect of pooling investment generations to achieve stability and to validate investment guarantees.

4. Segregated accounts [15]. This development was motivated by a desire to achieve greater flexibility in investing funds, primarily pension assets, than could be achieved within the legal and institutional frame work fashioned for managing investments generated by fixed dollar individual contracts. The declining importance attached to interest rate guarantees in pension fund administration and the related elevation of yield as a primary objective has been an important factor in the pension field for several years.

5. The entry of life insurance companies into mutual funds. The motivating factors for this action were probably a desire to recapture a larger share of the savings of individuals and to more fully and economically utilize existing sales forces. In any case, the decision means that life insurance company investment managers will have more funds to invest which do not carry any guarantees.

It is perfectly obvious that for the indefinite future, life insurance company investment managers will be handling assets to match large blocks of liabilities that carry some sort of investment performance guarantees. However, there has been a remarkable development of services and products offered by life insurance companies that will generate investment funds that match liabilities that do not carry such guarantees. For these new types of products and services, the investment objectives appear to be significantly different from those associated with traditional individual life insurance operations. An ever increasing proportion of the work of life insurance investment managers will be in handling funds where yield rather than stability is an overriding objective.

A question which remains open in the actuarial design of the new equity linked products is whether any investment risk should be assumed by the insurance company and, if so, what should be the precise nature of the guarantee [9, 32]. If death or withdrawal benefits at specific points or intervals of time are guaranteed, an additional risk is created for the insurance company. To manage this risk and to put a price on the granting of a benefit guarantee, a structure must be created to determine the range of the possible costs and to attach probabilities to these costs. There are investment management implications associated with such benefit guarantees. The premium to compensate the insurance company for absorbing the risk associated with a guarantee must be invested to match the risk. That is, funds to insure against swings in the equity market should not be invested in the same securities to which the guaranteed benefit amounts are linked.

Summary

In this essay an attempt has been made to trace the interaction between the actuarial plan for insurance products and the implied investment policy for the assets generated to match the insurance liabilities. Until very recently in the history of life insurance in the United States, considerable stress was placed on fixed dollar guarantees for death, withdrawal, and income benefits. The liabilities associated with these guarantees were rather naturally matched by long term fixed dollar investments. In recent years concomitant with inflation and economic growth there seems to have been a declining interest on the part of the purchasers of life insurance products and services for fixed dollar guarantees. This apparent market reaction has spawned a variety of new products and services in which investment results are reflected more directly in benefit payments.

No conjecture has been ventured as to the future course of these tendencies and preferences. Nor have their social implications been judged, although they may be awesome. However, if present trends continue, the impact on the life insurance industry will be great, and the impact on the capital markets will also be considerable. If insurance purchasers no longer favor guaranteed benefits perhaps securities with floating interest rates or price indexed bonds may be required to encourage savers to use insurance and annuity systems.

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